Lecture 12. **Integral quality estimation techniques**

Fourth group of quality criterions contains integral criteria, which give estimation of some generalized quality properties of a system, such as accuracy, stability margin and speed of response.

Integral techniques aimed to provide general estimation of decay speed and displacement value in aggregate, without defining their exact values separately. Equation (4.9) can serve as simple integral estimation:

$$J_1 = \int_0^\infty \varepsilon(t)dt \quad . \qquad (4.9)$$

Here $\varepsilon(t)$ is the displacement in time.

In a stable system $\varepsilon(t) \to 0$ as $t \to \infty$ and hence the (4.9) integral is finite.

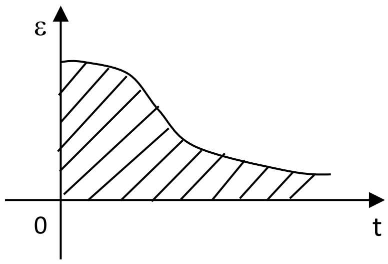

Fig. 4.12. Monotonous transient

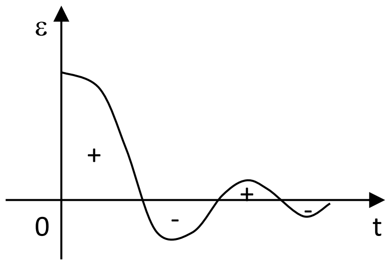

Fig. 4.13. The oscillating transient

Geometrically this integral is obviously equal to the area under the displacement transient curve (fig. 4.12 and 4.13). The smaller the area, the faster this transient decays and the smaller is the value of displacement. So, it is recommended to adjust system parameters in a way that makes this integral the smallest possible.

Equation (4.9) has some inconvenience since it is applicable to monotonous processes only, when the displacement value always has the same sign $\varepsilon(t)$.

But if we have oscillating process, another integral is more appropriate:

$$J_2 = \int_0^\infty |\varepsilon(t)| dt \qquad (4.10)$$

which is the sum of absolute values of areas under the curve.

But it figured out in practice that analytical calculation is often difficult!

In light of this it is more convenient to compute square integral estimation, sometimes called “square area” (fig. 4.14):

$$J_3 = \int_0^{\infty} \varepsilon^2(t)dt \qquad (4.11)$$

This estimation is independent of displacement sign change, and hence of transient behavior (ho matter, whether it is monotonous or oscillating one).

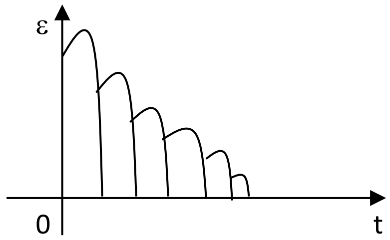

Fig. 4.14. Displacement squared

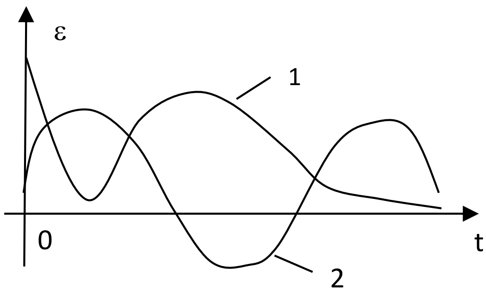

Fig. 4.15. Displacement oscillating transient

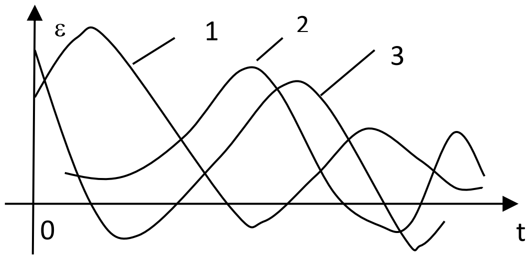

Fig. 4.16. Different transient behaviors

The value of the criterion J_3, the less is the sum of areas (taken for the squares of ordinates) the less will be J_3 (fig. 4.14).

Integral (4.11) is also called quadratic dynamic control error. If written dimensionless, it is:

$$J_2 = \frac{\Omega_0}{c^2} \int_0^{\infty} \varepsilon^2(t)dt \quad . \qquad (4.12)$$

Here *ε(t)* is a deviation from stable value, *c* is some constant having dimension of controlled variable, and Ω_0 is geometric mean of the characteristic equation roots (*).

In general, disadvantage of integral criteria is that they have nothing to do with the form of transient curve. It works out transients with completely different forms (fig. 4.16) have analogous integral estimations (4.11). Also it is not uncommon to have excessively oscillating system when parameters are set according to minimal integral value. The reason for this is the integral estimation counts only numerical value of displacement and decay speed, and not checks at all whether the system is in the vicinity of oscillatory stability threshold.

To fix this issue another integral approach is used: count not only ε(t), but the speed of change too, i.e. $\dot{\varepsilon}(t)$. Improved square integral has the form:

$$J_4 = \int_0^\infty (\varepsilon^2 + T\dot{\varepsilon}^2)dt, \qquad (4.13)$$

where *"T"* is time constant.

Corresponding transient is presented graphically in fig. 4.17, and has the form:

$$h(t) = h_0 e^{-\frac{t}{T}} + (1 - e^{-\frac{t}{T}}). \qquad (4.14)$$

So it is possible to make transient look like exponent with time constant T by adjusting system parameters in order to obtain the minimal integral value (4.13).

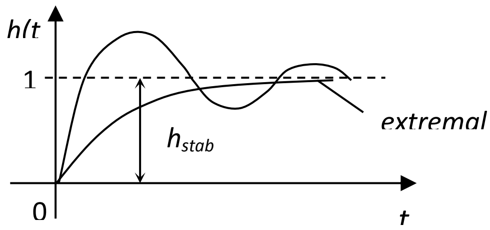

Fig. 4.17 Transient curve

Of course, we can further improve integral criterion by including derivatives of higher orders.

$$J_4 = \int_0^\infty (\varepsilon^2 + T_1^2\dot{\varepsilon}^2 + T_2^2\ddot{\varepsilon}^2)dt.$$

Here we should not forget about economic expediency.

To conclude, integral criteria give a single number assessing the quality of a system, and it is their main advantage. On the other hand, one and the same integral value can correspond to completely structurally different transient behaviors, what makes the task solution unclear.